

**Midtoets Complexe Analyse**  
**15/12/11, 09.00–11.00 uur**

1. Define the function  $f(z)$  by

$$f(z) = x^2 + iy^2, \quad z = x + iy.$$

- (a) Is this function analytic on  $\mathbb{C}$ ? Explain your answer.  
(b) Prove that  $f$  is differentiable on the line  $x = y$ .

2. Consider the function  $f(z) = \cos z$  on  $\mathbb{C}$ .

- (a) Show that the zeros of  $f$  are real, and determine all zeros.  
(b) Use the Cauchy-Riemann equations to prove that  $f$  is an entire function.  
(b) Is  $f$  bounded on  $\mathbb{C}$ ? Explain your answer.

3. Compute the integral

*forward* 
$$\int_{\Gamma} \frac{dz}{z-1},$$

where  $\Gamma$  is the ~~half~~ circle parametrized by  $z(t) = 1 + re^{it}$ ,  $0 \leq t \leq \frac{1}{2}\pi$ . Here,  $r$  is a given positive constant.

4. Use the Cauchy integral theorem to compute the integral

$$\int_C \frac{e^{-z}}{z^2 + 4} dz,$$

where  $C$  is the circle  $|z - 2i| = 2$  traversed once in counter-clockwise direction. Explain your answer.

5. Consider the function  $f(z) = (z - 2)(z - 3)$  on the set

$$V = \{x + iy \in \mathbb{C} : 0 \leq x \leq 1, 0 \leq y \leq 1\}.$$

- (a) Why does  $|f(z)|$  attain its maximum  $M$  on  $V$ ?  
(b) Compute  $M$ .

Explain your answer.

Points: 1: 9 + 9, 2: 6 + 6 + 6, 3: 18, 4: 18, 5: 9 + 9. 10 for free.